

Thermodynamic analysis of the V-type Stirling-cycle refrigerator

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Abstract

The thermodynamic analysis of a V-type Stirling-cycle refrigerator is performed. The Stirling-cycle refrigerator consists of expansion and compression spaces, cooler, heater and regenerator, and divided into 14 fixed control volumes subjected to a periodic mass flow. The conservation of mass and energy equation are written for each control volume. A computer program is prepared in FORTRAN, and the basic equations are solved iteratively. The mass, temperature and density of working fluid in each control volume are calculated for a given charge pressure, engine speed, and fixed heater and cooler surface temperatures, and the results are obtained from a PC. The heat transfer coefficients are assumed constant. The work, instantaneous pressure and COP of the Stirling-cycle refrigerator are also calculated. The steady cyclic conditions are obtained for temperature after few cycles and the results are given by diagrams.

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Keywords: Thermodynamic cycle; Stirling; Modelling; Software; Calculation; Performance; COP

Refrigérateur Stirling de type V: analyse thermodynamique

Mots clés : Cycle thermodynamique; Stirling; Modélisation; Logiciel; Calcul; Performance; COP

1. Introduction

A Stirling-cycle machine operates on a closed regenerative thermodynamic cycle using a working gas, and subjects the gas to expansion and compression processes at different temperatures. Since Stirling engines are externally heated, they can be powered using a wide variety of fuels and heat sources. Despite these benefits, the Stirling engine

is not widely used due to the high manufacturing costs, working fluid leakage. On the other hand, the Stirling-cycle refrigerator has no ozone depletion potential.

The first machine was built by R. Stirling in 1816 as an alternative to the steam engine, and other machines developed, are different in detail. The classical analysis of Stirling-cycle machine is given by Schmidt, and in the literature considerable efforts have been made to develop and improve this analysis. Schmidt assumed sinusoidal volume variations for the expansion and compression spaces and uniform pressure through the system.

Tew et al. [1] analyzed the thermodynamic characteristics of the rhombic-drive and a free-piston Stirling-cycle engine, and simulated by software. The model represents the

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Nomenclature

A	Heat transfer area (m^2)	ρ	Density (kg m^{-3})
COP	Coefficient of performance	θ	Crank angle (rad)
h	Heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)	Subscripts	
i	Specific enthalpy (J kg^{-1})	c	Compression volume property
m	Mass (kg)	CV	Control volume
p	Pressure (Pa)	d	Dead volume
Q	Amount of heat transfer (J)	e	Expansion volume property
R	Gas constant ($\text{J kg}^{-1} \text{K}^{-1}$)	in	Into the control volume
T	Absolute temperature (K)	i	Belongs to i th control volume
U	Internal energy (J)	out	Out of control volume
V	Volume (m^3)	s	Swept volume
V_s	Swept volume (m^3)	t	Total
W	Amount of work done (J)	w	Wall property
ϕ	Phase angle of the compression space relative to the expansion space variation (rad)		

working space by a series of subdivisions, which is called the nodal model. Urieli and Kushnir [2] have shown that this analysis can be utilized in order to evaluate the various practical effects of non-ideal regenerators, heat exchangers, including heat transfer and pressure losses. Ataer [3] and Urieli and Berchowitz [4] presented an extensive review of Stirling cycle machines.

Berchowitz and Unger [5] presented the performance of free-piston Stirling coolers designed for domestic refrigeration. The engine uses helium as the working fluid and has the potential for long life and low noise. The performance is shown competitive when compared to vapor compression systems. Berchowitz and Schonder [6] presented the calculations to design of an integrated Stirling cooler using natural gas as the fuel and helium as the working fluid. Berchowitz [7] presented a simple linear analysis of the dynamics of free-piston Stirling coolers to describe the behavior of these machines. Results are obtained for the displacer phase angle and stroke ratio, and it is shown that the linear analysis is able to accurately account for the dissipative losses in a given machine. This analysis has been used to design a number of successful prototype machines including small electronic coolers and a 100 K, 200 W lift cryocooler.

Ataer [8,9] used Lagrangian method on the analysis of regenerators of Stirling-cycle machines. The governing equations of the regenerator are derived in terms of the displacement of the displacer, so that time does not appear in the equations. The equations, which include the pressure fluctuations due to flow reversals and longitudinal conduction, are solved numerically by a digital computer using a finite difference method.

By discussing the irreversibilities Berchowitz [10] optimized a free-piston Stirling-cycle refrigerator with

reference to designing machines for operation at intermediate temperatures. Walker et al. [11] gave the brief review of the previous work for compact Stirling refrigerators.

Blanck et al. [12] made an optimal power analysis of an endoreversible Stirling cycle with perfect regeneration and obtained maximum power and efficiency at maximum power for the cycle based upon higher and lower temperature bounds. Their results provide additional criteria for use in the study and performance evaluation of Stirling engines. Wu et al. [13] performed the analysis of a Stirling engine with heat transfer and imperfect regeneration irreversibilities, and derived the relationship between the net power output and thermal efficiency of the engine.

Ladas and Ibrahim [14] presented a finite-time thermodynamic analysis of the Stirling-cycle engine based on mass and energy balances. The governing equations are solved numerically, and the effect of cycling rate and regeneration on power output and efficiency studied. They have shown that there exists an optimum power output for a given engine, based on cycling rate.

Angelino and Invernizzi [15] have shown that the heat pumps based on Stirling cycle are positively influenced by real gas effects, provided they are designed to operate in a proper region of the fluid state diagram. Kaushik and Kumar [16] also presented an investigation of a finite-time thermodynamic analysis of an endoreversible Stirling engine to maximize the power output and the corresponding thermal efficiency of an endoreversible Stirling heat engine with internal heat loss in the regenerator and for the finite heat capacity of the external reservoirs.

In the present study the control volume analysis of the V-type of Stirling-cycle refrigerator is performed. The refrigerator is divided into 14 control volumes as shown in Fig. 1. The refrigerator consists respectively of a

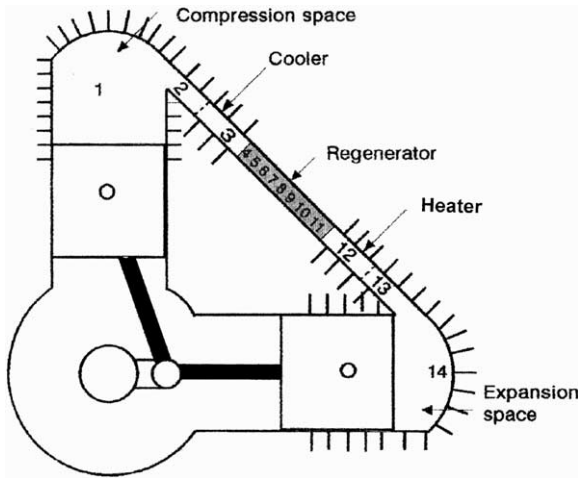


Fig. 1. Schematic of the Stirling-cycle refrigerator.

compression space, 1; cooler, 2, 3; regenerator, 4–11; heat absorber, 12, 13; and expansion space, 14. Heat is transferred to the refrigerant in the expansion space and heater, and transferred to the environment in the compression space and cooler.

When the air is flowing from expansion space to the compression space, the half-cycle is called compression half-cycle, and the air is flowing from compression space to the expansion space during the expansion half-cycle. During the compression half-cycle heat is transferred from the metal matrix to the refrigerant in the regenerator, and during the expansion half-cycle to the metal matrix in the regenerator.

Each control volume in the V-type refrigerator is an open system subjected to a periodic mass flow. The control volumes of compression and expansion spaces are variable volumes, and the volumes of 2–13 have fixed volumes. The conservation of mass and energy equation are written for each control volume of the refrigerator. A computer program is written in FORTRAN, and the equations are solved iteratively.

2. Theory

The variable expansion and compression spaces; 14 and 1 of the V-type Stirling-cycle refrigerator are expressed as respectively

$$V_e = V_{e,d} + \frac{1}{2} V_s (1 + \cos \theta) \quad (1)$$

and

$$V_c = V_{c,d} + \frac{1}{2} V_s [1 + \cos(\theta - \phi)] \quad (2)$$

where $V_{h,d}$ and $V_{c,d}$ are the dead volumes of the expansion and compression volumes respectively, and assumed equal.

ϕ is the phase angle of the compression space relative to the expansion space and taken as 90° in the analysis.

The equation of state and energy equation are the basic equations of each control volume. The mass balance for the control volume of 'i' is written as

$$m_{i,in} - m_{i,out} = \Delta m_{i,CV} \quad (3)$$

The total mass of the V-type Stirling-cycle refrigerator is constant and equal to

$$m_1 + m_2 + \dots + m_n = m_t \quad (4)$$

where n is the number of the control volumes and taken as 14 in the analysis. The energy equation for the control volume in the general form shown in Fig. 2, is written as:

$$Q_i - W_i = (m_i)_{i,out} - (m_i)_{i,in} + (\Delta U)_{i,CV} \quad (5)$$

This equation is written for each control volume. Within the working temperature and pressure intervals of the refrigerator, air may be assumed as an ideal gas [17]. The equation of state is

$$p = \rho RT \quad (6)$$

It is assumed that there is no pressure drop in the Stirling-cycle refrigerator, and the instantaneous pressure is uniform. Using Eq. (6) in Eq. (4) the instantaneous pressure of the refrigerator is written as

$$p = \frac{m_t R}{\frac{V_1}{T_1} + \frac{V_2}{T_2} + \frac{V_3}{T_3} + \dots + \frac{V_n}{T_n}} \quad (7)$$

where the subscripts indicate the control volume of the V-type Stirling-cycle refrigerator.

The COP of the V-type Stirling-cycle refrigerator is defined as

$$\text{COP} = Q/W \quad (8)$$

where Q and W are the amount of cooling and work done per cycle on the refrigerator, respectively.

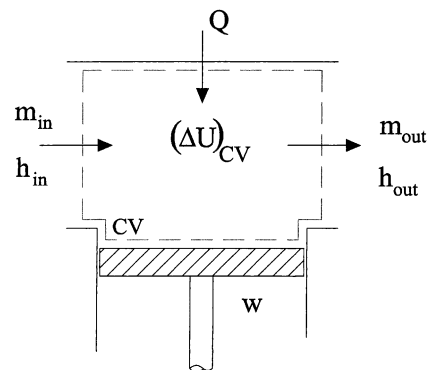


Fig. 2. The general form of the energy equation for the control volume.

3. Numerical method

The V-type Stirling-cycle refrigerator consists of an expansion space, heater, regenerator, cooler, compression space, expansion and compression space pistons and a slider-crank mechanism as shown in Fig. 1. In the numerical analysis, the mass, temperature and density of working fluid in each control volume are calculated for a given charge pressure, engine speed, and fixed heater and cooler surface temperatures. The work and COP of the Stirling-cycle refrigerator are also calculated for a given charge pressure, engine speed, pressure, and fixed heater and cooler surface temperatures.

The heat transfer coefficients in the V-type refrigerator are taken constant; 300 W/m² K for expansion and compression spaces, 150 W/m² K for the heater and cooler and 600 W/m² K for the regenerator.

For the thermodynamic analysis the V-type Stirling-cycle refrigerator is divided into 14 control volumes as indicated in Fig. 1. The first control volume of the refrigerator is the compression space; 2, 3 are cooler; 4–11 are regenerator; 12, 13 are heater; and 14 is the expansion space. Each control volume is considered as an open system subjected to a periodic mass flow. The control volumes of 2–13 have fixed volumes, and the compression and expansion spaces have variable volumes. A computer program is prepared in FORTRAN, the conservation of mass and energy equation are solved iteratively.

In the analysis the surface temperatures of the heater, cooler, expansion and compression volumes are assumed to be constant. It is also assumed that the regenerator-matrix temperature distribution is linear in axial direction and constant with time. This is an acceptable assumption and in agreement with the results proposed by Ataer [8,9].

The following additional assumptions are made in order to simplify the analysis:

- The total mass of the working fluid is constant within the refrigerator.
- The working gas is air and can be treated as an ideal gas.
- The angular speed of Stirling-cycle refrigerator is constant.
- The cyclic conditions are established within the system.
- It is assumed that the pressure losses due to friction are negligible.
- The specific heats of the working gas are constant.
- The gas flow in the duct is one dimensional, and there is no radial temperature variation within the control volume.

A simplified iterative numerical method is developed to solve the equations of the V-type Stirling-cycle refrigerator. Analysis is started at the beginning of expansion half cycle when the expansion space piston is at the top dead point and the compression space piston is moving upward. At these positions of the pistons the instantaneous pressure of the

refrigerator is calculated using Eq. (6) and assuming the gas temperature is equal to the wall temperature. Then, the density and mass of the gas in each fixed control volume is calculated.

After the $\Delta\theta=0.5^\circ$ interval of crank angle the densities of the air in each control volume are calculated, and then using the new densities and previous temperatures of the control volumes, the instantaneous pressure of the refrigerator is calculated again.

On the next step using the new pressure the density of the gas is calculated for each control volume. At last step the temperature of the first control volume is calculated by

$$T_1^{\theta+\Delta\theta} = T_1^\theta + \Delta T_1 \quad (9)$$

By this the first cycle of the first control volume is completed. For each case using the similar method the new temperature of the control volume 1 and previous temperatures of other control volumes p , ρ_i , m_i , T_1 are calculated up to their values reach to a constant values.

4. Numerical results

Air is used as the working gas, and the thermodynamic analysis of the Stirling-cycle refrigerator is performed for different charge pressures. The compression and expansion space initial gas temperatures are taken as 320 and 260 K, respectively. The results are obtained using the parameters given in Table 1.

A computer program is prepared in FORTRAN and the results are obtained from a PS. The mass, temperature and density of working fluid in each control volume are calculated in addition to the refrigerator pressure iteratively. Periodic steady cyclic conditions are obtained after few cycles.

The variation of temperatures of the control volumes 1, 2, 3, 8, 12, 13 and 14 with crank angle at periodic steady-cyclic conditions are shown in Fig. 3. The variation of non-dimensional mass of the hot and cold spaces, and variation of non-dimensional total volume with crank angle periodic steady-cyclic conditions are given in Fig. 4. The mass flow rates in the expansion and compression volumes should be equal, and this condition requires larger dead volume in the

Table 1
Data used on the analysis of the Stirling-cycle refrigerator

Angular speed, (rpm)	1000
Charge pressure, (bar)	2
Displacement of pistons, (cm)	5
Piston diameter (cm)	5
Phase angle (degree)	90
Total mass of the working gas (g)	0.682
Swept volume per cylinder (cm ³)	98.175
Minimum total volume (cm ³)	195.755
Maximum total volume (cm ³)	334.595

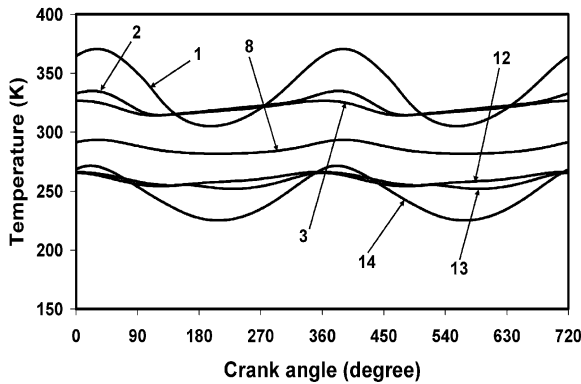


Fig. 3. Variation of temperature of fixed control volumes with crank angle at periodic steady-cyclic conditions.

expansion space. Otherwise, the performance of the Stirling-cycle refrigerator will be low. This will also affect the amount of regenerative heat exchange. Fig. 4 shows that the assumed magnitudes of dead volumes in the analysis are satisfactory.

The P–V diagram of the Stirling-cycle refrigerating machine for the charge pressure of 2 bar is given in Fig. 5. In comparison, the result obtained from the Schmidt analysis is also given in the same figure. For this study the amount of work done and cooling rate of the machine are 3.05 and 4.37 J per cycle, respectively. Under the same conditions the Schmidt analysis gives a work of 1.39 J per cycle that is 51% lower than this analysis.

Variation of COP and wall temperature of expansion space with the charge pressure is shown in Fig. 6. As seen in the figure COP decreases as the charge pressure increases. This is due to increase in the cyclic work. On the other hand the expansion space wall temperature also decreases with charge pressure. Variation of the cooling rate and work per cycle with charge pressure at the periodic steady cyclic conditions are given in Fig. 7.

The surface areas of the expansion volume (control volume of 14), and the heater (control volumes of 12, 13) are important parameters. In the analysis heat transfer

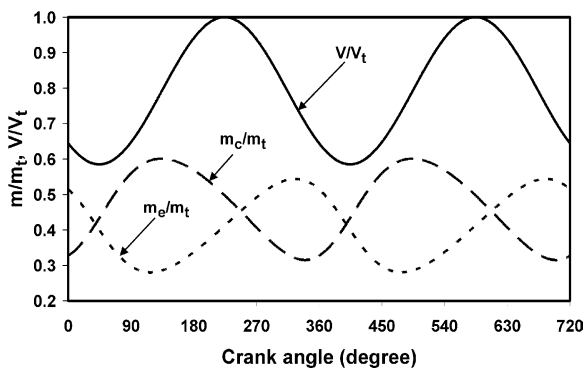


Fig. 4. At periodic steady-cyclic conditions variation of m_c/m_t , m_g/m_t and V/V_t with crank angle.

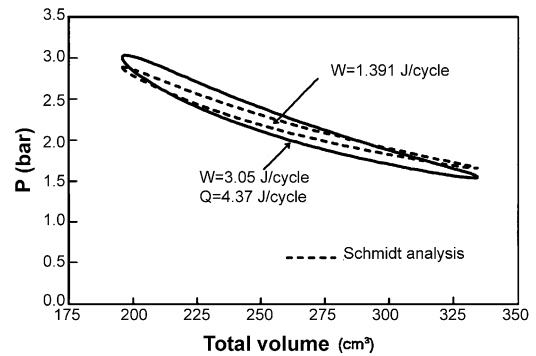


Fig. 5. P–V diagram of the Stirling-cycle refrigerator for 2 bar charge pressure.

coefficients are needed, and the product of heat transfer area and heat transfer coefficient; (A_h) may be considered as an independent parameter. The difference between the gas and the wall temperatures is the basic criteria on the determination of (A_h). The differences between the wall and gas temperatures at the highest velocity of the working fluid are shown in Fig. 8. Figure shows that the A_h is estimated adequately. The variation of cooling rate, cycling work and the A_c/A_h ratio with the heat transfer area of the heater is shown in Fig. 9.

5. Conclusions

The control volume analysis provides the information for the comparison of several aspects of the Stirling-cycle refrigerator. In this study air is used as a working gas. The charge pressure of the Stirling-cycle refrigerating machine is taken between 2 and 5 bar. The charge pressure higher than 2 bar will decrease the COP of the Stirling-cycle refrigerator.

As shown in Fig. 1, 14 control volumes are considered in the analysis. For precise results number of control volumes should be high.

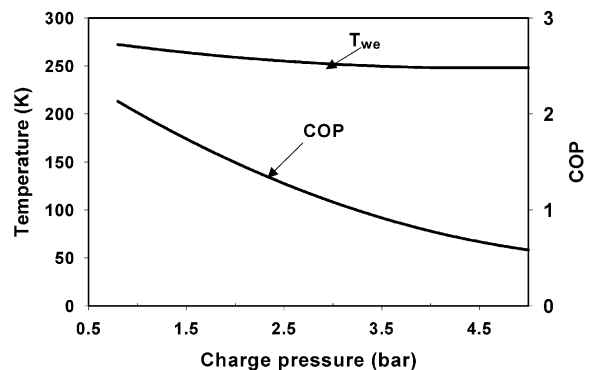


Fig. 6. Variation of the COP, and the wall temperature of the expansion space with the charge pressure.

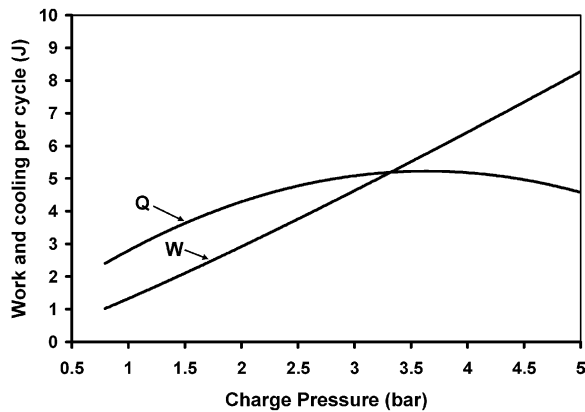


Fig. 7. Variation of the cyclic cooling and work with charge pressure.

Increase in the heat transfer areas and heat transfer coefficients (A_h) causes increase in the COP of the machine.

In the analysis instantaneous pressure of Stirling-cycle refrigerator is taken as constant and defined by Eq. (6). Research is going on the simulation of the refrigerator considering the pressure drop in each control volume.

The following conclusions are derived from the analysis:

1. The analysis provides the necessary data for the comparison of several aspects of the Stirling-cycle refrigerator.
2. At near-ambient cooling conditions the COP of the V-type Stirling-cycle refrigerating machine is comparable with the vapor pressure refrigerating systems.
3. The Stirling-cycle refrigerator can be designed using the results of this control volume analysis.
4. For domestic cooling 2 bar charge pressure and the compression ratio of 1.5–2.0 is appropriate.
5. Increase on the heat transfer area and heat transfer coefficients (A_h) causes increase in the COP of the refrigerator.

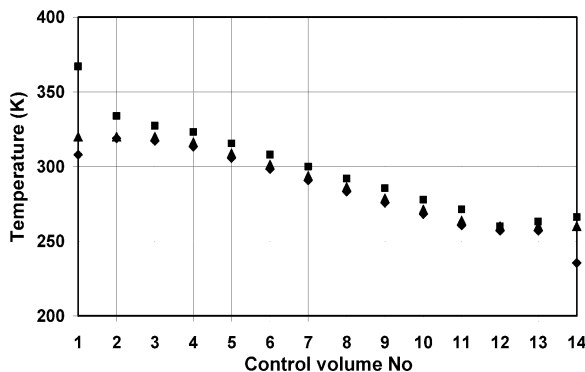


Fig. 8. Instantaneous control volume temperatures at the highest velocity of compressed and expanded gas flow (■ Expanded gas flow, ♦ Compressed gas flow, ▲ Surface temperature).

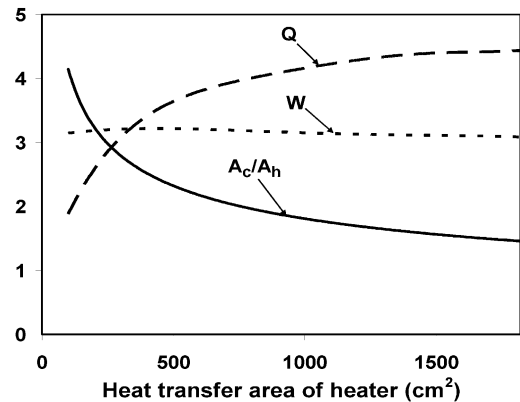


Fig. 9. Variation of work, cyclic cooling and A_c/A_h ratio with the heat transfer area of heater.

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